B.Com Sem – II
Business Mathematics

UNIT – III

'Simple and Compound Interest'

Introduction :

When a person borrows money, he pays it back to the money lender after some specified period of time. Also he pays some extra money for the privilege having used the money.

That extra money he pays is called Interest.

The money he borrowed is called Principal.

The money (in total) he pays to the lender after specified time period is called Amount.

⇒ Money paid at the end of period = Money Borrowed + Extra Money

⇒ $\boxed{\text{Amount} = \text{Principal} + \text{Interest}}$

Generally Used Notations :

P – Principal
A – Amount
SI – Simple Interest
CI – Compound Interest.
t – Time for which principal is borrowed
r – Rate of Interest

Rate of Interest : ($r\%$)

The interest paid on principal of ₹100 for one period (period may be year, half year etc.)

Types of Interest :

1) Simple Interest
2) Compound Interest
3) Continuous Compound Interest.

Simple Interest :

When the interest is calculated only on the principal initially invested for whole the period of use, it is called Simple Interest.

Formula to calculate SI :

$$SI = \frac{P \times r \times t}{100}$$

Example :

Calculate SI paid for a principal of 10,000/- at the end of 3 years at 5% per annum.

Given : $P = ₹\,10,000$
$r = 5\%$
$t = 3$ yrs

$$SI = \frac{P \times r \times t}{100} = \frac{10,0\cancel{00} \times 5 \times 3}{1\cancel{00}}$$

$$= ₹1500.$$ Ans

Example :

At what rate of interest the principal of ₹10,000 will yield simple interest of rupees ₹1500 in 3 years?

$r = ?$

$P = ₹10,000$

$SI = 1500$

$t = 3$ yrs

$$SI = \frac{P \times r \times t}{100}$$

$$1500 = \frac{10,0\cancel{00} \times r \times 3}{1\cancel{00}}$$

$$r = \frac{15\cancel{00}}{1\cancel{00} \times 3} = \frac{15}{3} = 5\%$$

$$\therefore r = 5\% \quad \underline{\text{Ans}}$$

Example :

Veena deposited ₹ 7200 in a finance company which pays 15% interest p.a. Find the interest and the amount she is expected to get after 4 years and 6 months.

$P = ₹ 7200$

$r = 15\%$

$t = 4\frac{1}{2}$ yrs $= \frac{9}{2}$ years

$$SI = \frac{P \times r \times t}{100} = \frac{72\cancel{00} \times 15 \times \frac{9}{2}}{1\cancel{00}} = ₹4,860$$

and

$$\text{Amount} = P + SI = 7200 + 4860$$

$$= ₹ 12,060$$

Compound Interest :

It is the interest obtained on the money accumulated at the end of each period.

Accumulated money means Principal + Interest of the earlier periods.

Formula to calculate CI :

$$\text{Amount after } n \text{ periods } (= A) = P \times \left(1 + \frac{r}{100}\right)^n$$

ie, $\boxed{A = P\left(1 + \frac{r}{100}\right)^n}$, n is no. of periods.

$$\Rightarrow \quad CI = \text{Amount} - \text{Principal}$$

$$\boxed{CI. = P\left(1 + \frac{r}{100}\right)^n - P}$$

Depreciation at Compound Rate :

If there is depreciation at the rate of $r\%$ compounded (eg., depreciation of a machine etc), then the formula for value V after n periods of depreciation becomes :

$$\boxed{V = P \times \left(1 - \frac{r}{100}\right)^n}$$

Note :

If the rate of interest for different periods are different, say $r_1, r_2, r_3, \ldots, r_k$ are the rates of interest for $n_1, n_2, \ldots, n_k$ period resp., then

$$\boxed{A = P \times \left(1 + \frac{r_1}{100}\right)^{n_1} \times \left(1 + \frac{r_2}{100}\right)^{n_2} \times \text{------} \times \left(1 + \frac{r_k}{100}\right)^{n_k}}$$

Example :

Find the compound interest on ₹16,000 at 15% p. a. for 3 years.

$P = ₹\,16,000$

$r = 15\%$

$t = 3$ yrs

$$A = P\left(1 + \frac{r}{100}\right)^t$$

$$= 16000\left(1 + \frac{15}{100}\right)^3 = 16000\left(\frac{115}{100}\right)^3$$

$$= 16000\left(\frac{23}{20}\right)^3$$

$$= \overset{2}{\cancel{16000}} \times \frac{23}{\cancel{20}} \times \frac{23}{\cancel{20}} \times \frac{23}{\cancel{20}}$$

$$= 24,334 \text{ ₹}$$

and $CI = A - P$

$CI = 24,334 - 16,000$

$= ₹\,8,334$ <u>Ans</u>

Example :

Find the CI for 2 years & 6 months on a sum of ₹ 2,60,000 at rate of 16% compounded annually.

$P = ₹2,60,000$

$r = 16\%$ p.a $= 8\%$ half yearly

$t = 2$ yr and $\frac{1}{2}$ year.

$$\therefore A = P\left(1+\frac{r}{100}\right)^t$$

$$= 2,60,000\left(1+\frac{16}{100}\right)^2\left(1+\frac{8}{100}\right)^1$$

{∵ when time is half, then reduce the rate to half and take time period one.}

$$= 26,0000 \times (1.16)^2 \times (1.08)$$

$$= 377844.48 ₹$$

$$\therefore CI = A - P$$

$$= 377844.48 - 2,60,000$$

$$= ₹\ 1,17,844.48$$ Ans

Note :

(i) $\log xy = \log x + \log y$

(ii) $\log \frac{x}{y} = \log x - \log y$

(iii) $\log x^n = n \log x$.

Example :

Find the compound interest on ₹ 6950 for 3 years if interest is payable half yearly. The rate of interest for first two years being 6% p.a and for the third year is 9% per annum.

$P = ₹\,6950$

For first 2 years, $r_1 = 6\%$ p.a
$= 3\%$ half yearly

$t = 2$ years $\Rightarrow n_1 = 4$ half years.

For third year, $r_2 = 9\%$ p.a $= 4.5\%$ half yearly
and $n_2 = 2$ half years.

{∵ If rate p.a. is compounded half yearly, then reduce the rate to half and double the time period.}

$$\therefore A = P\left(1 + \frac{R}{100}\right)^t$$

$$A = 6950\left(1 + \frac{3}{100}\right)^4\left(1 + \frac{4.5}{100}\right)^2$$

$$A = 6950 \times (1.03)^4 \times (1.045)^2$$

Taking log on both sides,

$$\log A = \log 6950 + 4 \log 1.03 + 2 \log 1.045$$
$$= 3.8420 + 4(0.0128) + 2(0.0191)$$
$$= 3.9314$$

Taking anti-log on both sides,

$$A = \text{Anti-log}(3.9314)$$

$$A = 0.8539 \times 10^4$$
$$A = 8539$$

$$\therefore CI = A - P$$
$$= 8539 - 6950$$
$$= ₹1589 \quad \underline{\text{Ans}}$$

Exercise :

Q-1 At what rate % p.a will ₹ 1600 amount to ₹ 1933 in 2 years if the interest is compounded half yearly?

Q-2 The simple and compound Interest on a certain sum for two years are ₹ 40 and ₹ 41 respectively. Find the rate and sum.

Q-3 A Machinery plant costing ₹ 10,000 depreciates each year by 10% of its value at the beginning of the year. After how many years will it be valued at half of its original value?

Q-4 The population of a village is 24,000. It is increasing at the rate of 5% every year. What will be the increase or growth in the population after 3 years?

Q-5 A sum on compound interest becomes ₹2420 in 2 years and ₹ 2662 in 3 years. Find the rate and the sum.

'Continuous Compound Interest'

If the rate of interest is $r\%$ p.a. and the interest is compounded continuously, then after n no. of years,

the amount A for the principal P is $Pe^{\frac{r \times n}{100}}$.

i.e., $\boxed{A = Pe^{\frac{r \times n}{100}}}$

$e = 2.718$ (approx.)

Example:
Find the effective rate of interest of 9% p.a. compounded continuously.

Let $P = 100$
$r = 9\%$

$A = 100 \times e^{\frac{9 \times 1}{100}}$

$= 100 \times (2.7183)^{0.09}$
$= 109.4$

Interest $= 109.4 - 100 = 9.4$

$\therefore$ Effective rate of Interest $= 9.4\%$

Example:

Find the nominal rate of interest when it is compounded continuously is equivalent to the effective rate of interest of 6.2% p.a.

The effective rate of interest ₹6.2 is the actual interest earned on ₹100 in 1 year.

$\therefore$ $P = 100$

$A = 106.2$.

Let rate $= r\%$

$$A = P e^{\frac{r \times n}{100}}$$

$$106.2 = 100 \; e^{\frac{r \times 1}{100}}$$

$$106.2 = 100 \; (2.7183)^{r/100}$$

Taking log on both sides

$$\log 106.2 = \log 100 + \left(\frac{r}{100}\right) \log (2.7183)$$

$$2.0261 = 2 + \left(\frac{r}{100}\right) 0.4343$$

$$\left(\frac{r}{100}\right) 0.4343 = 0.0261$$

$$r = \frac{0.0261 \times 100}{0.4343}$$

$$r = 6.0097$$

$\therefore$ $\boxed{r \simeq 6\%}$ Ans.

'Nominal And Effective Rate of Interest'

Nominal rate of Interest :

The given interest rate of which compounding frequency is more than once a year is called Nominal Interest Rate.

Eg: The annual rate 6% converted quarterly is annual nominal rate.

Effective Rate of Interest :

It is the actual interest earned on ₹100 in 1yr.

Formula to calculate Effective rate for given nominal rate (r%) of compounding period n per year,

$$\boxed{\text{Effective rate} = 100 \times \left(1 + \frac{r}{100}\right)^n - 100}$$

Example :

Let ₹100 be invested for a year at 10% rate compounded half yearly, then the amount after 1 year will be

$$A = 100 \times \left(1 + \frac{5}{100}\right)^2 = 110.25$$

$$CI = 110.25 - 100 = 10.25$$

⇒ The Actual interest earned on ₹ 100 for 1 year is ₹ 10.25 on nominal rate of 10%.

So, effective rate of interest is 10.25%

Example :

Find the effective rate of interest of 8% p.a. compounded quarterly.

$r = 8\%$ p.a. $= \frac{8}{4}\%$ quarterly $= 2\%$ quarterly.

Time = 1 year = 4 quarters.

$\Rightarrow$ Amount for ₹100 after one year $= 100 \times \left(1 + \frac{2}{100}\right)^4$

$= 108.24$

Interest after 1 year $= 100 - 108.24 = 8.24$

$\therefore$ Effective rate of Interest $= 8.24\%$.

Exercise

1) Find the effective rate of interest of 6% p.a. compounded continuously.

2) Find the effective rate of interest of 7.5% p.a. compounded continuously.

3) Which is better investment ; 12% p.a. compounded quarterly or 12.2% p.a. compounded continuously ?

4) Find the effective rate of interest of 12% p.a. compounded (i) quarterly
   (ii) monthly.

Business Mathematics
B.Com Sem-II

UNIT-III

Annuties

Annuity :
A sequence of payments of equal amount made at equal intervals of time for a specific period.

The person who pays instalment of an annuity is called Annuitator.
The person who receives the payment is called Annuitant or Annuity holder.

Annuities

- Annuity Certain
  - Ordinary or Immediate
  - Due
  - Deferred.
- Annuity Contingent

Types of Annuities : (i) Annuity certain
(ii) Annuity Contingent

Types of Annuity Certain : (i) Annuity Immediate/Ordinary
(ii) Annuity Due
(iii) Annuity Deferred

Annuity Certain :
If the time period for which instalments of an annuity are to be made is fixed then it is called Annuity Certain.

Eg : The payment of loan of a house / car.
Recurring deposits in Banks.

Annuity Contingent :
If some conditions are imposed on the payments of instalments or the instalments are made till the happening of some specific event then the annuity is called a Contingent Annuity.

Eg : The payments made till the completion of education of a child etc.

Annuity Immediate / Ordinary Annuity :
If the payments of instalments are made at the end of each period, the annuities are called annuity immediate.

'The payment fall at the end of each period.'

Annuity Due :
If the payments of instalments are made at the beginning of each period, the annuities are called annuity due.

'The payment fall at the beginning of each period

Deferred Annuity:

If we let the money accumulate for some time and then annuity starts, the annuity is called Deferred annuity.

"The instalments of the annuity are started after the lapse of some fixed time."

Bonds:

A bond is also money at maturity.

Sinking fund:

A fund created by setting aside a fixed amount periodically and allowing it to accumulate at compound interest.

Debenture issue and sinking fund are money at maturity ie., Amount.

Amount of an Annuity:

Total worth of all the payments of an annuity under consideration at the time of conclusian of the annuity.

The amount of annuity is the sum of principal values of the payments plus the interest of all the payments.

Formula to find Amount of an Annuity :

Let 'a' be the installment of an annuity per period made for 'n' periods at the rate of r% per period.

Let $i = \frac{r}{100}$.

(i) If the annuity is ordinary or immediate, then the formula for amount of annuity is :

Amount : $\boxed{A = a\left[\frac{(1+i)^n - 1}{i}\right]}$

(ii) In case the annuity is due the formula for amount is :

Amount : $$A = a\left[\frac{(1+i)^n - 1}{\left(\frac{i}{i+1}\right)}\right]$$

$$\boxed{A = a(1+i)\left[\frac{(1+i)^n - 1}{i}\right]}$$

Example

Find the amount of an annuity of ₹ 100 per year paid at the end of each year for 5 years at the rate 3% compounded annually.

Installement, $a = 100$ ₹

No. of payments, $n = 5$

$r = 3\% \Rightarrow i = \frac{r}{100} = \frac{3}{100} = 0.03$

$$A = a\left[\frac{(i+1)^n - 1}{i}\right] \quad \left\{\because \text{The annuity is an ordinary annuity.}\right\}$$

$$= 100\left[\frac{(1+0.03)^5 - 1}{0.03}\right] = 100\left[\frac{(1.03)^5 - 1}{0.03}\right]$$

Let $x = (1.03)^5$

$\log x = 5 \log 1.03$

$= 5(0.0128) = 0.0640$

Take Antilog :

$x = 0.1152 \times 10^1 = 1.159$

$\Rightarrow (1.03)^5 = 1.159.$

$$A = 100\left[\frac{1.159 - 1}{0.03}\right]$$

$$= 100 \times \frac{0.159}{0.03}$$

$$= 100 \times 5.3$$

$$= 530$$ Ans

Example

Find the amount of an annuity due of ₹400 payable quarterly for 10 years at 8% compound interest.

$a = ₹400$ per annum

$= ₹100$ quarterly

$r = 8\%$ p.a. $= 2\%$ quarterly

$i = \frac{r}{100} = \frac{2}{100} = 0.02$

$n = 10 \times 4 = 40$

Let Amount $= A$.

Since the annuity is due:

$$A = 100 \times (1+i)\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= 100 \times 1.02 \times \left[\frac{(1.02)^{40} - 1}{0.02}\right]$$

Let $x = (1.02)^{40}$

$\log x = 40 \log 1.02 = 40 \times (0.0086) = 0.3440$

$x = \text{anti}\log 0.3440 = 2.208$

$\Rightarrow (1.02)^{40} = 2.208$

$$\therefore A = 100 \times 1.02 \times \left[\frac{2.208 - 1}{0.02}\right]$$

$$A = \frac{100 \times 1.02 \times 1.208}{0.02}$$

$A = 6160.80$.

Exercise

1. Calculate the amount of an annuity immediate of ₹ 2,000 for 10 years, if the rate of interest be 5% per annum.

2. Find the amount of an ordinary annuity of 12 monthly payments of ₹ 1,000 that earn an interest at 12% per year compounded monthly.

3. Calculate the future amount of an annuity of ₹ 40,000 payable at the end of each quarter for 6 years at 8% p.a. compounded quarterly.

4. A company sets aside a sum of ₹ 50,000 annually for 10 years to pay off a debenture issue of ₹ 6,00,000. If the sinking fund is accumulated at 5% p.a. compound interest, find the surplus after full redemption of the debenture issue.

5. Find the amount of an annuity of ₹ 200 paid at the beginning of each 6 months for 20 years at 2% per annum compounded semi annually.

Present Value

Worth of all the installments made, at the time of beginning of an annuity is called the present value of the annuity.

Eg: Suppose the installments made for a T.V. of cost ₹10,000 purchased is 'a' for 'n' periods at the rate of r%, then the worth of all these payments at the time of purchase of the T.V. is 10,000. Hence present value of the annuity is ₹ 10,000.

Formula

Let 'a' be the instalment of an annuity per period paid for n periods at rate r% per period, $i = \frac{r}{100}$ be the rate per rupee.

P - Present value

A - Amount of all the payments.

(i) The present value of an annuity immediate of ₹ 'a' per period at r% per period for n periods.

$$\boxed{P = a \times \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]}$$

or $$P = a \times \left[\frac{1 - (1+i)^{-n}}{i}\right]$$

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(ii) The present value of an annuity due of ₹ 'a' per period at r% per period for n periods is:

$$\boxed{P = a \times (1+i)\left[\frac{(1+i)^n - 1}{i \times (1+i)^n}\right]}$$

or $$P = a \times \left[\frac{1-(1+i)^{-n}}{\frac{i}{1+i}}\right]$$

Example

Find the present value of an annuity of ₹100 per year at the end of each year for 5 years at 3% compounded annually.

$a = 100$

$r = 3\%\ p.a \Rightarrow i = 0.03$

$t = 5\,yr \Rightarrow n = 5$

$\therefore$ Present value, $P = a\left[\frac{1-(1+i)^{-n}}{i}\right]$

$$= 100 \times \left[\frac{1-(1+0.03)^{-5}}{0.03}\right]$$

$$= 100\left[\frac{1-(1.03)^{-5}}{0.03}\right]$$

$$= 100\left[\frac{1-0.8630}{0.03}\right] \quad \left\{\because (1.03)^{-5} = 0.8630\right\}$$

$$= 100 \times \frac{0.137}{0.03}$$

$$= 456.67 \quad \text{Ans.}$$

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Example

A man borrows ₹ 30,000 at 12% p.a. compound interest from a bank and promises to pay off the loan in 20 annual payments beginning at the end of first year. What is the amount of the annual payment? Given $(1.12)^{20} = 9.638$.

Present value $P = 30{,}000$

$r = 12\%$ p.a. $\Rightarrow i = 0.12$

$n = 20$

The annuity is an ordinary annuity.

Let the instalment $= a$

$$\therefore \quad P = a\left[\frac{1-(1+i)^{n}}{i}\right]$$

$$30{,}000 = a\left[\frac{1-(1+0.12)^{-20}}{0.12}\right]$$

$$= a\left[\frac{1-(1.12)^{-20}}{0.12}\right]$$

$$= a\left[\frac{1-\frac{1}{(1.12)^{20}}}{0.12}\right]$$

$$= a\left[\frac{(1.12)^{20}-1}{0.12 \times (1.12)^{20}}\right]$$

$$= a\left[\frac{9.638-1}{0.12 \times 9.638}\right]$$

$$\Rightarrow \quad a = \frac{30000 \times 0.12 \times 9.638}{8.638}$$

$a = ₹4016.76$ <u>Ans</u>

Example :

A loan of ₹ 30,000 is to be paid in equal instalments payable in the begining of each year in 20 years. Find the amount of each instalment if the rate of interest be 4% p.a.

The payments are made in the beginning of each period, so the annuity is the annuity due.

Let Instalment = a

$r = 4\% \Rightarrow i = 0.04$

$n = 20$

Present value of the annuity is $P = 30,000$

$$P = a\left[\frac{1-(1+i)^{-n}}{\frac{i}{1+i}}\right]$$

$$30,000 = a\left[\frac{1-(1+0.04)^{-20}}{\frac{0.04}{1.04}}\right]$$

$$30,000 = a \times \frac{104}{4} \times \left[1-(1.04)^{-20}\right]$$

$$30,000 = a \times 26 \times (1 - 0.4571)$$

$$= a \times 26 \times 0.5429$$

$$\Rightarrow a = \frac{30,000}{26 \times 0.5429}$$

$$\Rightarrow a = 2125.34$$ **Ans**

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Exercise

1. Find the present value of an annuity immediate of ₹ 60 per year payable half yearly for 8 years at 10% p.a. compounded semi-annually.

2. Find the present value of an annuity due of ₹ 1000 for 14 years allowing interest at 9% p.a.

3. A man borrowed ₹ 2000.5 at 6% p.a. CI promising to pay ₹ 500 at the end of each of the first 4 years and reduce the principal and interest and to pay the balance at the end of 5th year. Find the amount of his final payment.

4. Find the present value of an ordinary annuity of ₹ 60 per year payable half yearly for 8 years at 4% p.a. payable half yearly?

Deferred Annuities

Amount of a deferred Annuity :

$$A = a\left[\frac{(1+i)^n - 1}{i}\right]$$

Amount of a deferred annuity, deferred for 'm' periods, of instalment 'a' paid for 'n' periods at the rate of CI r% per period is same as the amount of an ordinary annuity at the same rate and time for 'n' periods. It is so because the period 'm' for which there is no transaction of money, have no role to affect the amount.

Present Value of a Deferred Annuity :

Present value $P = a\left[\frac{(1+i)^{-m} - (1+i)^{-(m+n)}}{i}\right]$,

where m is the deferred period.

Perpetuities (Perpetual Annuities) :

These are payments (or cash flows) of equal amounts that are made every period, for an unlimited no. of periods.

Eg: Property tax payments, preferred stocks.

Amount and Present value of an Annuity when the Interest is Compounded continuously :

Let 'a' be the instalment of annuity per period made for n periods at the rate of r% p.a. compounded continuously; then

Amount, $A = a\int_0^n e^{it}\,dt$

$$\boxed{A = \frac{a}{i}\left[e^{ni} - 1\right]}$$

Present Value is,

$P = a\int_0^n e^{-it}\,dt$

$$\boxed{P = -\frac{a}{i}\left(e^{-n\times i} - 1\right)}$$

where $i = \frac{r}{100}$.

Example

Find the amount and present value of a sequence of annual payments of ₹ 6,000 each, the first being made at the end of 5 years and the last at the end of 12 years. The money is worth 6% p.a. effective.

The deferred period of the annuity is 4 years as the payments for fifth years onward are being made. Since the sequence of money is paid for 12−4 = 8 years. So,

$$a = 6000 \quad , n = 8 \quad , r = 6\% \Rightarrow i = 0.06$$

$$A = a\left[\frac{(1+i)^n - 1}{i}\right] = 6000 \times \left[\frac{(1.06)^8 - 1}{0.06}\right]$$

$$= 6000 \times \left[\frac{1.593 - 1}{0.06}\right]$$

$$= 6000 \times \frac{0.593}{0.06}$$

$$= 59,300.$$

⇒ Amount of the deferred Annuity = ₹ 59,300.

Also, $m = 4, n = 8 \Rightarrow m + n = 12$

$$P = a\left[\frac{(1+i)^{-m} - (1+i)^{-(m+n)}}{i}\right]$$

$$= 6000 \times \left[\frac{(1+0.06)^{-4} - (1+0.06)^{-12}}{0.06}\right]$$

$$= 6000 \times \left[\frac{\frac{1}{(1.06)^4} - \frac{1}{(1.06)^{12}}}{0.06} \right]$$

$$= 6000 \times \left[\frac{(1.06)^8 - 1}{0.06 \times (1.06)^{12}} \right]$$

$$= 6000 \left[\frac{1.593 - 1}{0.06 \times 2.012} \right]$$

$$= 6000 \times \frac{0.593}{0.06 \times 2.012}$$

$$= 29{,}473.16.$$

$\Rightarrow$ Present Value is $P = ₹\ 29{,}473.16$.

Example :

A bank pays interest at the rate of 6% p.a. compounded Continuously. Find how much should be deposited in the bank each year in order to accumulate ₹ 6,000 in 3 years? ($\log e = 0.4343$)

Amount $= 6000$, $r = 6\% \Rightarrow i = 0.06$

$n = 3$

Let annual instalment $= a$

$$A = a \int_0^n e^{it}\, dt = \frac{a}{i}\left[e^{ni} - 1 \right]$$

$$\Rightarrow 6000 = \frac{a}{0.06}\left[e^{0.06 \times 3} - 1 \right]$$

$$\Rightarrow 6000 \times 0.06 = a\left[e^{0.18} - 1 \right]$$

Let $x = e^{0.18}$

$\log x = 0.18 \log e$

$\log x = 0.18 \times 0.4343$

$\log x = 0.0782$

$x = \text{anti-}\log(0.0782)$

$x = 1.198.$

$\Rightarrow e^{0.18} = 1.198$

$\therefore 360 = a \times (1.198 - 1)$

$a = \frac{360}{0.198}$

$a = 1818.18$ **Ans**

Example :

A man deposits ₹ 6000 every year for 2 years in an account earning 6% interest p.a. compounded continuously. What is the present value ? ($\log 2.7183 = 0.4343$).

$a = 6{,}000$

$r = 6\% \Rightarrow i = 0.06$

$n = 2.$

$$\therefore P = a \int_0^n e^{-it}\, dt = \frac{-a}{i}\left[e^{-ni} - 1\right]$$

$$P = -\frac{6000}{0.06}\left(e^{-0.12} - 1\right)$$

$$P = \frac{6000}{0.06}\left(1 - 0.8869\right) = 11{,}310$$

$\therefore$ Present Value = ₹ 11, 310.

Exercise :

1) Find the present value of an annuity of ₹ 10,000 p.a. at the rate of 8% p.a. compounded continuously for 2 years.

2) A man borrows ₹ 4,00,000 at 8% p.a. compounded continuously and agrees to pay both the principal and interest in 10 yearly equal instalments. Find the amount of each instalment.

3) Instalments of ₹ 5,000 are deposited every year in an account earning 4% p.a. interest compounded continuously. Find the amount of the annuity after 10 years. ($e^{0.4} = 1.491$).